Appendix This appendix is written by H.Shiga, H.Tanigawa, M.Toki and the author.

Our work began by studying Krushkal's paper [**7**], in which he proved strengthening pseudoconvexity (complex hyperconvexity) of the Bers embeddings T(Γ) ($\subset \mathbf{C}^m$) of finite dimensional Teichmüller spaces. That is, on T(Γ) there exists a plurisubharmonic function ρ(φ) (φ∈ T(Γ)) which tends to zero as φ goes to ∂T(Γ).

Krushkal' constructed such a function ρ(φ) in a concrete manner. In order to show ρ(φ) is upper semicontinuous, he relied upon the continuity of the embeddings of Teichmüller spaces in the universal Teichmüller space ([**7**] p.684, line 3 from the bottom). But, as has been seen in the text of this paper, we should do carefully such arguments. Thus, avoiding them, we will seek an elementary way to show the continuity of ρ(φ). So let us review briefly the function ρ(φ).

For $\varphi \in T(\Gamma) \subset A_2^\infty(\Gamma)$, we denote by f_φ the normalized quasiconformal automorphisms of the Riemann sphere $\hat{\mathbf{C}}$ which is conformal in { z | |z| > 1 }∪{∞} and whose Schwarzian derivative is φ. For Jordan curves C on $\hat{\mathbf{C}}$, we consider a functional

$$\lambda(C) = \left(\sup_{h \in H(C)} \frac{| D_G(h) - D_{\tilde{G}}(h) |}{D_G(h) + D_{\tilde{G}}(h)} \right)^{-1} = \frac{\max\left\{\kappa(G,\tilde{G}), \kappa(\tilde{G},G)\right\} + 1}{\max\left\{\kappa(G,\tilde{G}), \kappa(\tilde{G},G)\right\} - 1} .$$

Here, G and $\tilde{G}$ are the complementary simply connected domains divided by C, H(C) is the family of all functions h continuous in $\hat{\mathbf{C}}$ and harmonic in $G \cup \tilde{G}$, and $D_G(h)$ is the Dirichlet integral $\iint_G \left(h_x^2 + h_y^2\right) dxdy$, and

$$\kappa(G,\tilde{G}) = \sup_{h \in H(C)} \frac{D_G(h)}{D_{\tilde{G}}(h)} .$$

Now the definition of ρ is

$$\rho(\varphi) = \log\{\lambda(f_\varphi(\partial\Delta))^{-1}\}.$$

Then, the continuity of ρ(φ) follows from the next proposition which was implicitly in Schober [**10**] (or [**11**] under a similar condition). Thus Krushkal's argument becomes significantly short and clear, and his theorem is valid for the universal Teichmüller space, particularly for any Bers embedding of Teichmüller spaces. This hyperconvexity result itself has been announced in [**8**, Corollary 4] in a different way as an application of the Green function of Teichmüller spaces.

Proposition 2. *Let* f *be a quasiconformal automorphism of* $\hat{\mathbf{C}}$ *whose maximal dilatation is* K (≥1). *Then under the same notations as above,*

$$K^{-2}\kappa(G,\tilde{G}) \le \kappa(f(G),f(\tilde{G})) \le K^{2}\kappa(G,\tilde{G}) .$$

Proof. A continuous function on C determines the element of H(C) by the harmonic extensions to G and $\tilde{G}$. Thus the correspondence $h \mapsto h\circ f^{-1}$ of continuous functions on C to those on f(C) = C' determines the bijective map f_*: H(C)→H(C').

Since $D_{f(G)}(h\circ f^{-1}) \le KD_G(h)$ for $h \in H(C)$ and $D_G(h'\circ f) \le KD_{f(G)}(h')$ for $h'\in H(C')$, the Dirichlet principle implies that

$$D_{f(G)}(f_*(h)) \le KD_G(h) \quad \text{and} \quad D_G(f_*^{-1}(h')) \le KD_{f(G)}(h') .$$

Setting $h'=f_*(h)$, we have

$$K^{-1}D_G(h) \le D_{f(G)}(f_*(h)) \le KD_G(h) ,$$

and by the same reason,

$$K^{-1}D_{\tilde{G}}(h) \le D_{f(\tilde{G})}(f_*(h)) \le KD_{\tilde{G}}(h) \quad \text{for } \forall\, h \in H(C) .$$

Therefore we obtain the required inequality.
□